

# Application of Pulse Method to Incremental Slitting Measurement of Residual Stresses in Laminated Composites

M. M. Shokrieh<sup>1\*</sup>, Saeed Akbari<sup>2</sup>

Received: 5 May 2012; Accepted: 18 July 2012

**Abstract:** In this research, the incremental slitting method was employed to determine through-thickness residual stress profile of a carbon/epoxy laminate. The method involves measuring strains at the back surface of the stressed specimen, while a narrow slit is cut by a CNC milling machine progressively from the top surface of the specimen. "Pulse Method" was selected as the computational approach for the determination of residual stresses from measured strains. Results show that thermal residual stresses resulting from curing process of laminated composites have high values and may play a significant role in the failure of these components.

**Keywords:** Laminated Composites, Residual Stress, Incremental Slitting, Pulse method

## 1. Introduction

Residual stresses introduced during the curing process may have undesirable effects on the performance of composite components. For instance, they may lead to distortion and out-of-plane deformation of un-symmetric laminates, as shown in Fig. 1(a). Tensile residual stresses in the matrix are particularly important because they may represent a significant fraction of the tensile strength of the resin and can result in matrix cracking, as shown in Figure Fig. 1(b). The residual stress depends on the geometry of the laminate, the curing cycle and the mechanical properties [1]. It is essential that these stresses be determined in the design process, so that their possible influences on the safety and reliability of the composite components can be estimated. Otherwise, a higher safety factor should be considered, which results in an overdesigned structure.

Numerous relaxation methods have been employed by many researchers to measure residual stresses. In relaxation methods, the residual stresses are determined using the strain or deformation data due to the relief of stresses when some part of the stressed component is removed. The slitting method (or crack compliance method) is one of

these techniques that have been widely used for measuring residual stresses in different materials. This method was introduced and developed by Cheng and Finnie [2–5]. Prime [6] presented a survey on this method.

This work reports an application of the slitting method to determine residual stress distribution in a carbon/epoxy laminated composite. The pulse method was used as the computational approach to calculate the depth profile of the in-plane residual stress component normal to the slit face from the measured strains. Results of the slitting experiment show that thermal residual stresses in the composite parts have significant values and their possible effects on the component performance should be taken into account.

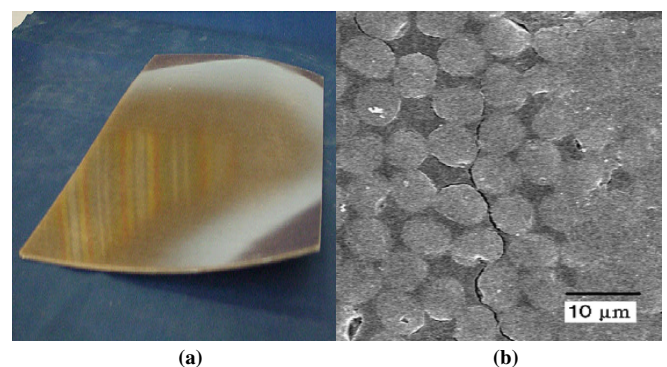


Fig. 1. Undesirable effects of residual stresses in laminated composites (a) out-of-plane deformation [7] and (b) matrix cracking [8].

1\*. Corresponding Author: Professor, School of Mechanical Engineering, Iran University of Science and Technology, Tehran, Iran (shokrieh@iust.ac.ir)  
2. M. Sc., School of Mechanical Engineering, Iran University of Science and Technology, Tehran, Iran (sakbari@mecheng.iust.ac.ir)

## 2. Principles of the slitting method

Over the past twenty five years, the slitting method has been used for residual stress determination in a vast variety of materials, such as stainless steel, functionally graded materials, metal matrix composites, Aluminum alloys, friction stir welds, etc. In this method, a narrow slit is progressively cut through the thickness of a stressed component. The released strains around the slit are measured using strain gauges bonded either on the top or back surfaces of the specimen. Residual stress distribution is then calculated using recorded strains and the compliance coefficients.

### 2.1. Background and terminology

Fig. 2 shows the typical geometry of the slitting method with a back surface strain gauge. This geometry includes the specimen thickness  $t$ , the specimen length  $L$ , the specimen width  $B$ , the slit depth  $a$ , the slit width  $w$  and the strain gauge length  $l$ . The strain gauges bonded on the back surface of the stressed part directly opposite the slit are sensitive to all residual stress within the specimen thickness and thus generally used for through-thickness measurements, but the strain gauges bonded on the top surface near the slit are only sensitive to near surface residual stress (usually up to 20-25% of the thickness) and therefore are appropriate for near surface measurements.

The slit starts from the top surface of the specimen and is extended in successive increments towards the back surface. For the configuration shown in Fig. 2, slitting method will determine unknown normal residual stress component perpendicular to the slit plane,  $\sigma_{yy}(x)$ , using  $y$ -strain measured by back surface or top surface strain gauges.

After recording released strains, the next step is to approximate the residual stress distribution from the measured strains data with an appropriate method. For this purpose, an initial distribution for residual stress must be considered. It is important to note that the form of initial stress distribution not only dictates how the compliance coefficients will be defined but also has a significant effect on the estimated residual stress. The most important

methods to estimate residual stress include "Series Expansion Method" and "Pulse Method" [9]. In the "Series Expansion Method", residual stress is approximated by a continuous polynomial with unknown coefficients. Because of discrepancy of material properties of different layers, the residual stress in laminated composites is discontinuous across layers boundaries. Therefore, "Series Expansion Method" is not applicable to composite laminates. For this reason, "Pulse Method" has been used in this research. A complete discussion of this method is presented in the next section.

### 2.2. Theoretical basis of the pulse method

In the relaxation methods of residual stress measurement, the relationship between the residual stresses and the measured strains or deformations does not have a simple one-to-one form. This is because the measured strains depend on all released stresses within the specimen thickness, and not just those at a specific depth. Obviously, for the top surface strain gauges, residual stresses near the surface have more effect on the measured strains. Therefore, for the methods based on incremental material removal the relationship between the residual stresses and the measured strain data has the form of an integral equation [9]. For the slitting method, this relationship is in the following form (Eq. 1):

$$\varepsilon_{yy}(A_i) = \frac{1}{E'} \int_0^{a_i} G(x, a_i) \sigma_{yy}(x) dx \quad (1)$$

where  $\varepsilon_{yy}(a_i)$  is the measured  $y$ -strain when the slit depth is  $a_i$ . The kernel function  $G(x, a_i)$  is equal to the measured strain due to a unit stress at depth  $x$  within a slit of depth  $a_i$ . In the slitting method, this function is usually obtained using a finite element method.

For a plate or beam specimen,  $E'$  is defined as Eq. (2) [10]:

$$\begin{cases} E' = E & \text{for } \frac{B}{t} \leq 0.5 & \text{plane stress} \\ E' = \frac{E}{1-\nu^2} & \text{for } \frac{B}{t} \leq 2 & \text{planestrain} \end{cases} \quad (2)$$

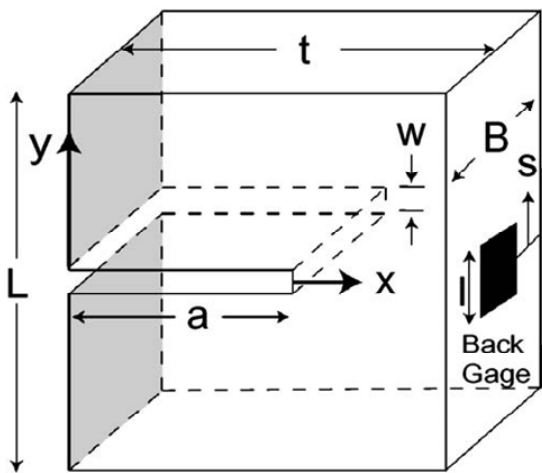


Fig. 2. Slitting method schematic.

In order to solve Eq. (1), an initial distribution for residual stress must be considered. In this research, "Pulse Method" was used for the approximation of residual stress. The main feature of the pulse method approximation is that it requires no explicit assumption for the residual stress distribution

Consider a residual stress profile acting on the faces of a slit of increasing depths  $a_1, \dots, a_i$ , as shown in Fig. 3 (a). From linear superposition the stress distribution can be estimated by a series of strip or pulse loads over each increment of the slit, denoted by  $\sigma_i$  and  $a_{i-1} \leq x \leq a_i$  ( $i = 1, \dots, n$ ), as shown in Fig. 3 (b). In other words, a uniform stress for each increment of slit depth is considered.

Therefore, in the "Pulse Method" residual stress is estimated by the Eq. (3).

$$\sigma(x_j) = \sum_{j=1}^n \sigma_j U_j(x) \tag{3}$$

where  $\sigma_j$  corresponds to the stress value in the  $j$ th increment. The pulse functions are defined as follows (Eq. 4):

$$U_j(x) = \begin{cases} 1 & a_{j-1} \leq x \leq a_j \\ 0 & x \leq a_{j-1}, \geq a_j \end{cases} \tag{4}$$

Substituting Eq. (3) in Eq. (1) results in:

$$\begin{aligned} \epsilon(a_j) &= \frac{1}{E'} \int_0^{a_j} G(x, a_i) \sum_{j=1}^n \sigma_j U_j(x) dx = \\ \frac{1}{E'} \sum_{j=1}^n \sigma_j \int_0^{a_j} G(x, a_i) U_j(x) dx &= \sum_{j=1}^n \sigma_j C_{ij} \end{aligned}$$

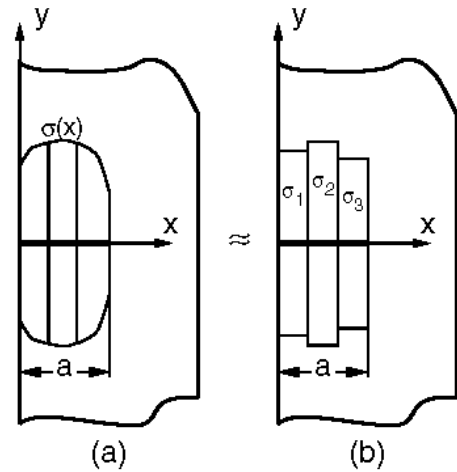


Fig. 3. (a) An unknown residual stress distribution on slit faces is approximated by (b) a series of uniform strip loads.

Therefore,  $C_{ij}$  or the elements of compliance matrix are expressed by the Eq. (5):

$$C_{ij} = \frac{1}{E'} \int_{a_{j-1}}^{a_j} G(x, a_i) U_j(x) \tag{5}$$

Comparing to Eq. (1) indicates that a specific element of the compliance matrix,  $C_{ij}$ , is the measured strain at the strain gage location for a slit of depth when residual normal stress distribution at the domain  $a_{j-1} \leq x \leq a_j$  is equal to the unit load (Eq. 6):

$$C_{ij} = \epsilon(a = a_i, \sigma(x) = U_j(x)) \tag{6}$$

Fig. 4 shows the physical interpretation of the compliance coefficients. There are analytical methods that can be used for calculating compliance coefficients in isotropic parts with simple geometries, such as plates, beams, cylinders, and disks, but a closed form solution is not yet available for orthotropic materials. In this work, these coefficients are calculated using finite element method described in [11].

### 3. Experimental procedure

The slitting method was used for the determination of residual stresses in a symmetric cross-ply carbon/epoxy laminate. A detailed description of the method of experimental procedure is presented in this section.

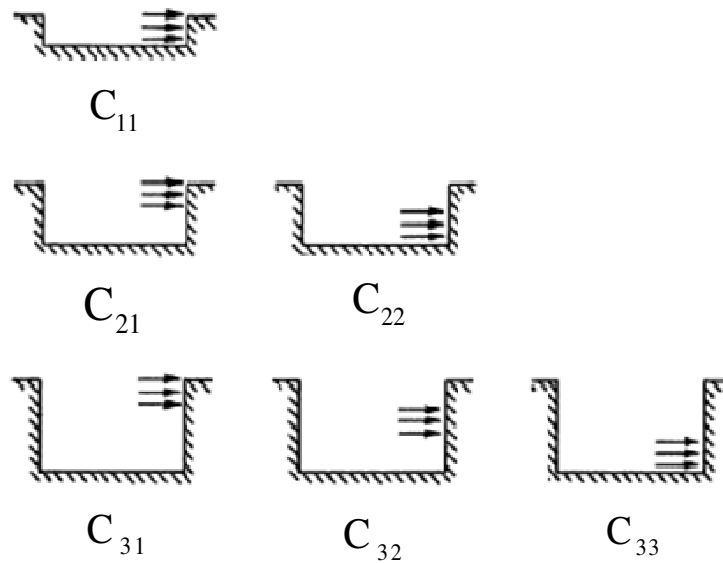


Fig. 4. Physical interpretation of compliance coefficients  $C_{ij}$ .

In this research,  $[0_4/90_4]_s$  carbon/epoxy laminate was fabricated using hand lay-up method. The material used was T300 unidirectional carbon fibers with epoxy resin ML-506 and hardener Aradure-830. This is a high performance composite used mostly in the aeronautical and aerospace industries.

The curing process involved temperature stages of 100 °C during 6 hour, followed by 120 °C for 6 hours. The heating and cooling rates were 4 °C/min. The Longitudinal tensile test, transverse tensile test and shear test was carried out to determine elastic constants. The results of the characterization tests are shown in Table 1. Fig. 5 shows the prepared specimens for the slitting experiment. Dimensions of these specimens are given in Table 2.

As mentioned in previous sections, the strain gauge can be bonded on the back surface of the specimen (back surface strain gage) or on the top surface near the slit (top surface strain gage). In such cases, the top surface gauge results are in error. This strain gauge should be as close as possible to the slit edge in order to record significant amounts of strain, but its results can be affected by yielding as the stresses relax during slitting. On the other hand, slitting process can introduce additional residual stress and affect the gauge results. Also, two component of residual

shear stresses released in the slit face can influence the recorded strains. Contrary to top surface strain gauge, back surface gauge is not subjected to these errors and its results are more reliable. For this reason, back surface gauge is used in this research. However, the disadvantage of back gauge is that it is less sensitive to near surface residual stresses.

Type UBFLA-03 strain gauge with a gauge length of 0.3 mm, supplied from TML Company, bonded to the back surface of specimen. In order to minimize the effect of averaging of the strain over the gauge length and to increase the precision of strain readings at the desired location, the gauge with smallest gauge length available among different types of gauges from different companies was selected.

Before bonding strain gauges, the surface of the specimens is not abraded; it is only degreased with acetone. This is because manual abrasion can change the residual stress state of the specimen. Relative position of strain gauge and cutter is shown in Fig. 6.

Table 1. Elastic constants of uni-directional carbon/epoxy ply

$E_x$ (GPa)	$G_{xy}$ (GPa)	$E_y$ (GPa) $\nu_{xy}$
104.6	3.8	7.5 0.31

Table 2. Dimensions of the composite specimens (mm)

Width (B)	Length (L)	Thickness (t)	Slit width (w)
16	62	4.82	0.252

The part was clamped from one side away from slit and gauge, as shown in Fig.6, so the other side could deform freely and recorded strains are correct. Slitting process was carried out in a CNC milling machine with a circular saw blade. The saw blade was 0.2 mm thick and 23 mm in diameter. Model MS-21XUSB of DSM digital data logger was used to record strains in depth increments of 0.3 mm.

#### 4. Results and discussion

Released strains measured during the slitting experiment are shown in Fig. 7. Fig. 8 shows the through-thickness residual stress distribution calculated using pulse method. Because of symmetry, only half of the specimen is considered in the stress calculations. The stresses are tensile through the 0° layers and compressive through the 90° layers. A local stress maximum occurs at the boundary of 0° / 90° layers.

It should be noted that there is a stress increase near the 0° / 90° boundary. An increase in the number of increments and a corresponding reduction in the depth of each increment, makes it possible to increase the sensitivity of the method and result in greater measuring precision, particularly near the 0° / 90° boundary.

The thermal residual stresses in laminated composites can also be determined analytically using classical lamination theory, but the theory results are not as precise as experimental results. For

instance, the theory is not able to determine the stress gradient for unidirectional layers [14].

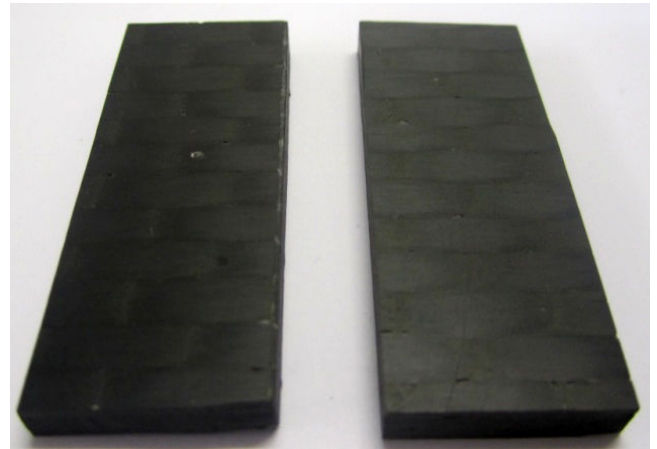


Fig. 5. Prepared composite specimens.

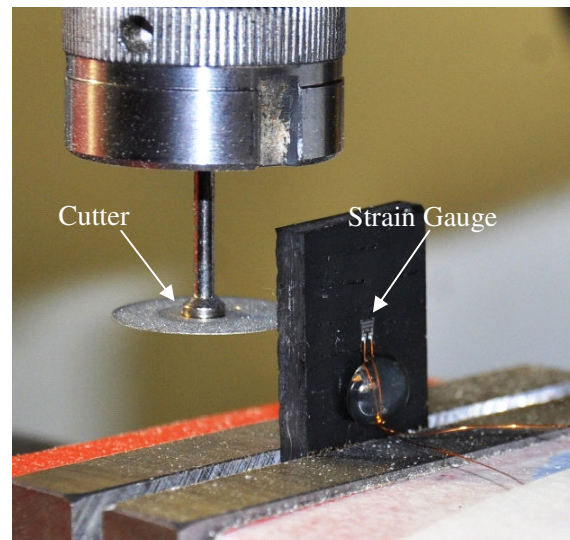


Fig. 6. Relative position of strain gauge and cutter in the slitting experiment.

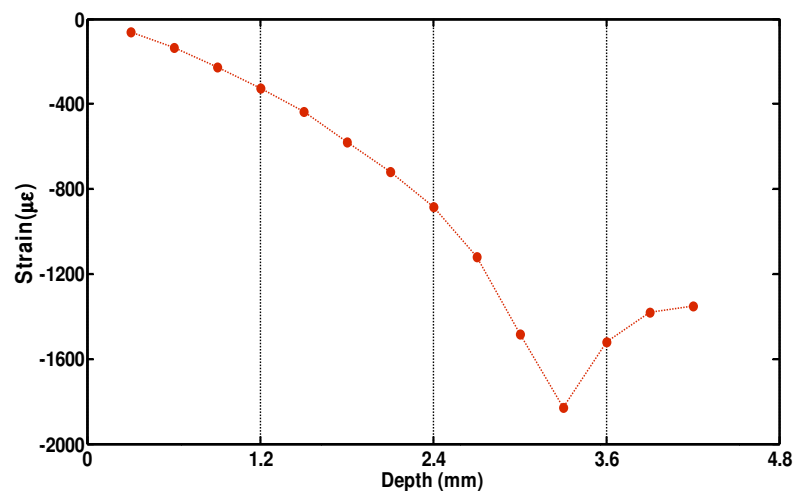


Fig. 7. Measured strains in slitting experiment.

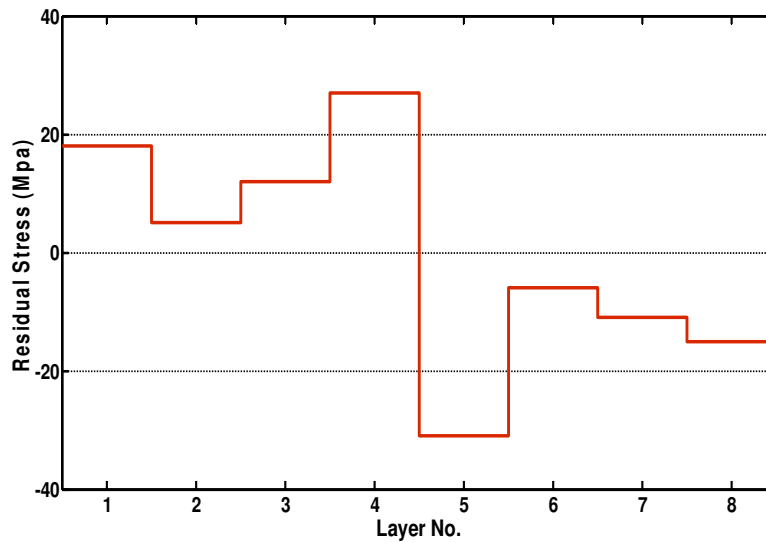


Fig. 8. Calculated stresses from the measured strains.

## 5. Conclusions

This research presents experimental measurements of the through-thickness distribution of residual stress in a carbon/epoxy laminated composite. The slitting method is employed to determine residual stress, where a slit is introduced using a CNC milling machine, and released strain is measured as a function of increasing slit depth. Back surface strain gauges are used, because they are experimentally much more robust to errors. With its incremental character, slitting method can determine the residual stresses in all layers of a composite laminate. Its principle is simple, and contrary to other existing experimental methods, it has the advantage to be easily practicable.

The "Pulse Method" was used as the computational approach for residual stress calculation. In this method, it is assumed that the residual stress has a constant value in each depth increment. The results show that residual stresses due to curing process have significant values and may adversely affect the mechanical behavior of composite.

## Reference

- [1] Kim, K-S.; Hahn, H.T.; Croman, R.B., "Effect of cooling rate on residual stress in a thermoplastic composite", *Journal of Composites Technology and Research*, 1989, 11(2), 47–52.
- [2] Cheng, W.; Finnie, I., "A Method for measurement of axisymmetric residual stresses in circum-ferentially welded thin-walled cylinders", *ASME Journal of Engineering Materials and Technology*, 1985, 107, 181–185.
- [3] Cheng, W.; Finnie, I., "measurement of residual hoop stress in cylinders using the compliance method", *ASME Journal of Engineering Materials and Technology*, 1986, 108, 87–92.
- [4] Cheng, W.; Finnie, I., "Deformation of an edge cracked strip subjected to arbitrary shear surface traction on the crack faces", *Engineering Fracture Mechanics*, 1992, 43, pp. 33–39.
- [5] Cheng, W.; Finnie, I., "The single slice method for measurement of axisymmetric residual stresses in solid rods or hollow cylinders in the region of plane strain", *ASME Journal of Engineering Materials and Technology*, 1998, 120, 170–176.
- [6] Prime, M.B., "Residual stress measurement by successive extension of a slot: the crack compliance method", *ASME Journal of Applied Mechanics*, 1999, 52(2), 75–96.
- [7] Sunderland P; Yu, W.; Mnson, J.-A., "A thermo-viscoelastic analysis of process-induced internal stresses in thermoplastic matrix composites", *Polymer*

- Composites, 2001, 22(5), 579–92.
- [8] Filiou, C. and Galiotis, C., "In situ monitoring of the fibre strain distribution in carbon-fibre thermoplastic composites- Application of a tensile stress field", *Composites Science and Technology*, 1999, 59(14), 2149–61.
- [9] Schajer, G.S.; Prime, M.B., "Use of inverse solutions for residual stress measurements", *ASME Journal of Engineering Materials and Technology*, 2006, 128(3), 375–382.
- [10] Lee, M. J.; Hill, M. R., "Effect of strain gage length when determining residual stress by slitting," *ASME Journal of Engineering Materials and Technology*, 2007, 129(1), 375–382.
- [11] M. M. Shokrieh; S. Akbari R., (2011), "Simulation of slitting method for calculation of compliance functions of laminated composites", *Journal of Composite Materials*, 2012, 46(9), 1101–1109.
- [12] Schajer, G. S., "Measurement of non-uniform residual stresses using the hole-drilling method. Part I. Stress calculation procedures", *ASME Journal of Engineering Materials and Technology*, 1988, 110, 338-343.
- [13] Schajer, G. S.; Prime, M.B., "Residual stress solution extrapolation for the slitting method using equilibrium constraints", *ASME Journal of Engineering Materials and Technology*, 2007, 129(2), 227-232.
- [14] Sicot, O.; Gong, X. L.; Cherouat, A.; Lu, J., "Determination of residual stress in composite laminates using the incremental hole-drilling method", *Journal of Composite Materials*, 2003, 37(9), 831–844.